

$$1a) f'(x) = x^2 + 1 + (x-1)2x = 3x^2 - 2x + 1 \quad (4P)$$

$$b) f'(x) = -\frac{1}{(1-x^2)^2} \cdot (-2x) = \frac{2x}{(1-x^2)^2} \quad (4P)$$

$$c) f'(x) = \frac{1}{(1+x^2)} - \frac{x}{(1+x^2)^2} \cdot 2x = \frac{1-x^2}{(1+x^2)^2} \quad (4P)$$

$$d) f(x) = \frac{(1-x^2)(1+x^2)}{1+x^2} = 1-x^2 \Rightarrow f'(x) = -2x \quad (4P)$$

$$e) f'(x) = e^{-x} - x e^{-x} = (1-x)e^{-x} \quad (4P)$$

$$f) f'(x) = \sin x + x \cos x \quad (4P)$$

$$g) f'(x) = \cos(x^2) + 2x^2 \sin(x^2) \quad (4P)$$

$$h) f'(x) = \ln|x| + x \cdot \frac{1}{x} = \ln|x| + 1 \quad (4P)$$

$$g) f'(x) = \frac{-1}{(x \ln|x|)^2} \cdot (x \ln|x|)' = \frac{-1}{(x \ln|x|)^2} (\ln|x| + 1) \quad (4P)$$

$$j) f(x) = \cos^2 x + \sin x \cdot \frac{\sin x}{\cos x} \cdot \cos x = 1 \Rightarrow f'(x) = 0 \quad (4P)$$

$$2a) \int f(x) dx = \int x^3 - x^2 + x - 1 dx = \frac{x^4}{4} - \frac{x^3}{3} + \frac{x^2}{2} - x + C \quad (4P)$$

$$b) \int \frac{1}{1-x^2} dx = \int \frac{1}{2} \left(\frac{1}{1-x} + \frac{1}{1+x} \right) dx = \frac{1}{2} (-\ln|1-x| + \ln|1+x|) + C \quad (4P)$$

$$1+x^2 = u, du = 2x dx$$

$$c) \int \frac{x}{1+x^2} dx \stackrel{1}{=} \int \frac{1/2 du}{u} = \frac{1}{2} \ln|u| + C = \frac{1}{2} \ln|1+x^2| + C \quad (4P)$$

$$d) \int f(x) dx \stackrel{cf. 1d)}{=} \int 1-x^2 dx = x - \frac{x^3}{3} + C \quad (4P)$$

$$e) \int x e^{-x} dx = x(-e^{-x}) - \int 1(-e^{-x}) dx = -x e^{-x} - e^{-x} + C \quad (4P)$$

$$f) \int x \sin x dx = x(-\cos x) - \int 1(-\cos x) dx = -x \cos x + \sin x + C \quad (4P)$$

$$g) \int x \cos(x^2) dx \stackrel{1}{=} \int \cos(u) \cdot \frac{1}{2} du \quad x^2 = u, du = 2x dx = \frac{1}{2} \sin(u) + C = \frac{1}{2} \sin(x^2) + C \quad (4P)$$

$$h) \int x \cdot \ln|x| dx = \frac{x^2}{2} \ln|x| - \int \frac{x^2}{2} \cdot \frac{1}{x} dx = \frac{x^2}{2} \ln|x| - \frac{1}{4} x^2 + C \quad (4P)$$

$$i) \int \frac{1}{x \ln|x|} dx \stackrel{1}{=} \int \frac{1}{u} du = \ln|u| + C = \ln|\ln|x|| + C \quad (4P) \quad u = \ln|x|, du = \frac{1}{x} dx$$

cf. 1.5)

$$i) \int f(x) dx = \frac{1}{2} \int 1 dx = x + C \quad (4P)$$

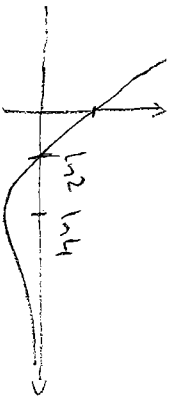
$$2a) f(x) = e^{-x}(2e^{-x}-1) \stackrel{!}{=} 0 \quad (2P)$$

$$\Rightarrow e^x = 2 \Rightarrow x = \ln 2 \quad (2P10)$$

$$f'(x) = -4e^{-2x} + e^{-x} = e^{-x}(1-4e^{-x}) \stackrel{!}{=} 0$$

$$\Rightarrow e^x = 4 \Rightarrow x = \ln 4 \quad (\text{maximum/minimum}) \quad (2P)$$

$$\lim_{x \rightarrow \infty} f(x) = 0 \quad \text{since } e^{-x}, e^{-2x} \rightarrow 0. \quad (2P)$$



(4P)

$$b) f(x) = e^{-x}(\cos x + \sin x) \stackrel{!}{=} 0$$

$$\Rightarrow \cos x + \sin x = 0 \Rightarrow \tan x = -1$$

$$\Rightarrow x = -\frac{\pi}{4} + k\pi \quad (\text{i.e. } -\frac{\pi}{4}, -\frac{\pi}{4} + \pi, -\frac{\pi}{4} + 2\pi, \dots) \quad (2P)$$

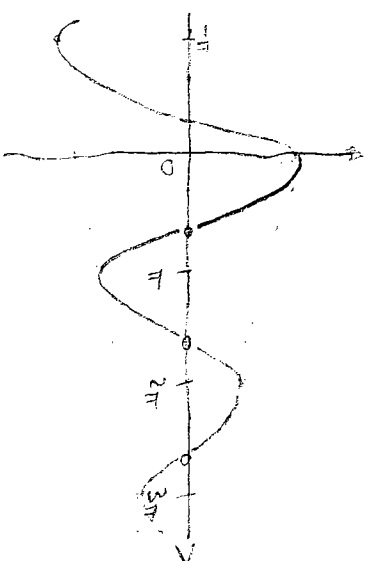
$$f'(x) = -e^{-x}(\cos x + \sin x) + e^{-x}(-\sin x + \cos x)$$

$$= -2e^{-x} \sin x \stackrel{!}{=} 0 \quad (2P)$$

$$\Rightarrow x = k\pi \quad (\text{i.e. } 0, \pi, -\pi, 2\pi, -2\pi, \dots) \quad (\text{maximum})$$

$$\lim_{x \rightarrow \infty} f(x) = 0 \quad \text{since } e^{-x} \rightarrow 0 \quad (2P)$$

and $\sin x, \cos x$ bounded



(4P)

$$A) f(x) = x^x = e^{\ln(x^x)} = e^{x \ln x} \quad (10P)$$

$$f'(x) = e^{x \ln x} \cdot (\ln x + x \cdot \frac{1}{x})$$

$$= e^{x \ln x} \cdot (1 + \ln x) \quad (10P)$$

$$= x^x (1 + \ln x)$$